Induced colorings of graphs and digraphs

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Let $G = (V, E)$ be a graph. A labeling $\varphi : V \cup E \to \mathbb{N} = \{1, 2, \dots\}$ is called a (K, L)-*total labeling*, if $\varphi(x) \leqslant K$ and $\varphi(uv) \leqslant L$, for all $x \in V$ and $uv \in E$. For every vertex $x \in V$, we define the *weight* of x by $w_\varphi(x) = \varphi(x) + \sum_{v \in N(x)} \varphi(xv)$. One may think of the function $w_\varphi : V \to \mathbb{N}$ obtained in this way, as of a vertex coloring *induced* by the labeling φ.

We consider the following general question: *What type of a graph coloring can be realized as an induced coloring with labels of bounded size?* In a positive case, we look for the least possible constants in a (K, L)-total labeling inducing a desired coloring of any graph. For example, it is known that every graph has a (K, L)-total labeling inducing the usual proper coloring with $(K, L) = (1, 5)$ and $(K, L) = (2, 3)$. It is conjectured that the same is true with $(K, L) = (1, 3)$ and $(K, L) = (2, 2)$ (these are the famous *1-2-3 Conjecture* and *1-2 Conjecture*, respectively). On the other hand, it is known that 2-*distance coloring* cannot be induced by a $(1, L)$-total labeling with any constant L.

In the talk we shall discuss the above "inducibility" issues for *majority*, *acyclic*, *star*, and *nonrepetitive* coloring of graphs, in both, directed and undirected version.

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