Independent Domination Subdivision in Graphs

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A set S of vertices in a graph G is a dominating set if every vertex not in S is adjacent to a vertex in S. If, in addition, S is an independent set, then S is an independent dominating set. The domination (independent domination) number $\gamma(G)$ ($i(G)$, resp.) of G is the minimum cardinality of a dominating (an independent dominating, resp.) set in G. The domination (independent domination) subdivision number $\mathrm{sd}_\gamma(G)$ ($\mathrm{sd}_i(G)$, resp.) is the minimum number of edges that must be subdivided (each edge in G can be subdivided at most once) in order to increase the domination (independent domination, resp.) number. We show [1] that for every connected graph G on at least three vertices, the parameter $\mathrm{sd}_i(G)$ is well defined and differs significantly from the well-studied domination subdivision number $\mathrm{sd}_\gamma(G)$. For example, if G is a block graph, then $\mathrm{sd}_\gamma(G) \leqslant 3$, while $\mathrm{sd}_i(G)$ can be arbitrary large. Further we show that there exist connected graph G with arbitrarily large maximum degree $\Delta(G)$ such that $\mathrm{sd}_i(G) \geqslant 3\Delta(G) - 2$, in contrast to the known result that $\mathrm{sd}_\gamma(G) \leqslant 2\Delta(G) - 1$ always holds. Among other results, we present a simple characterization of trees T with $\mathrm{sd}_i(T) = 1$.

References

[1] A. Babikir, M. Dettlaff, M.A. Henning, M. Lemańska, Independent Domination Subdivision in Graphs, *Graphs Combin.*37, 2021 pp.691-709.