Ramsey numbers of Boolean lattices

D.Grósz[(1)], A.Methuku[(2)], C.Tompkins[(3)]

[(1)] University of Pisa, Pisa, Italy
[(2)] University of Birmingham, Birmingham, UK
[(3)] Rényi Institute, Budapest, Hungary

The *poset Ramsey number* $R(Q_m, Q_n)$ is the smallest integer N such that any blue-red coloring of the elements of the Boolean lattice Q_N has a blue induced copy of Q_m or a red induced copy of Q_n. The *weak poset Ramsey number* $R_w(Q_m, Q_n)$ is defined analogously, with weak copies instead of induced copies.

Axenovich and Walzer[1] showed that $n + 2 \leqslant R(Q_2, Q_n) \leqslant 2n + 2$. Recently, Lu and Thompson[5] improved the upper bound to $\frac{5}{3}n + 2$. We solve this problem asymptotically by showing that $R(Q_2, Q_n) = n + O(n/\log n)$. Recent work of Axenovich and Winter[2] implies that the $n/\log n$ term is required.

In the diagonal case, Cox and Stolee[4] proved $R_w(Q_n, Q_n) \geqslant 2n + 1$ using a probabilistic construction. In the induced case, Bohman and Peng[3] showed $R(Q_n, Q_n) \geqslant 2n + 1$ using an explicit construction. Improving these results, we show that $R_w(Q_m, Q_n) \geqslant n + m + 1$ for all $m \geqslant 2$ and large n by giving an explicit construction.

References

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