

Homomorphisms and colourings of oriented graphs

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A homomorphism of a graph G to a graph H is a mapping from $V(G)$ to $V(H)$ which preserves the edges: $uv \in E(H)$ whenever $uv \in E(G)$. Graph homomorphisms generalize proper graph colourings since any proper k -colouring of G can be viewed as a homomorphism of G to the complete graph K_k . The chromatic number of G can thus be defined as the least integer k such that G admits a homomorphism to K_k .

By considering oriented graphs (digraphs without 2-cycles), and homomorphisms of oriented graphs (arc-preserving vertex mappings), we get a natural extension of proper colourings to oriented graphs. The oriented chromatic number of an oriented graph \vec{G} is then defined as the least integer k such that \vec{G} admits a homomorphism to some tournament \vec{T}_k of order k .

In this talk, we will survey results on homomorphisms and colourings of oriented graphs and propose some open problems. We will also discuss 2-dipath colourings of oriented graphs.

References

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